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Numerical Methods - MA 207
Numerical Solutions of Algebraic and Transcendental Equations

1. Find a root of the equation $x^3 - 4x - 9 = 0$, using bisection method correct to 3 decimal places.
2. Find the root of the equation $\cos x = x\, e^x$ using bisection method correct to 4 decimal places.
3. Find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places using iterative method.
4. Using fixed point iterative method, find a root of the equation $x^3 + x^2 - 100 = 0$ correct to 4 decimal places.
5. Apply method of iteration to find the negative root of the equation $x^3 - 2x + 5 = 0$ correct to 4 decimal places.
6. Find a real root of $2x - \log_{10} x = 7$ correct to 4 decimal places using iterative method.
7. Find the smallest root of the equation
$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \frac{x^5}{(5!)^2} + \cdots = 0.$$
8. Find the positive root of $x^4 - 4 = 0$ correct to 3 decimal places using Newton-Raphson method.
9. Find the root of the equation $x\, e^x = \cos x$ using the secant method correct to 4 decimal places.
10. Find a real root of the equation $x^3 - 2x - 5 = 0$ which lies between 2 and 3 by the method of false position correct to 3 decimal places.
11. Evaluate the following (correct to 4 decimal places) by Newton's iterative method.
    (a) $1/31$ (c) $1/\sqrt{14}$ (e) $30^{-\frac{1}{5}}$
    (b) $\sqrt{5}$ (d) $\sqrt[3]{24}$ (f) $22/7$.
12. Find the real root of the equation
$$x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \frac{x^{11}}{1320} \cdots = 0.4331135$$
correct to 4 places of decimals.
13. Using Newton-Raphson method, find correct to 4 decimal places, the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.
14. Find, by Newton's method, the root of the equation $e^x = 4x$, which is approximately 2, correct to 3 places of decimals.
15. Using Newton-Raphson method, establish the formula $x_{n+1} = \frac{1}{2}(x_n + \frac{N}{x_n})$ to calculate the square root of N. Hence find the square root of 5 correct to 4 places of decimals.
16. Show that the iterative formula for finding the reciprocal of N is $x_{n+1} = x_n(2 - Nx_n)$ and hence find the value of $1/31$.
17. Apply Newton's formula to find the root of $x^4 - x = 10$, which is nearer to $x = 2$.

18. Solve the equation $x = \frac{1}{2} + \sin x$ using iterative methods.

19. The equation $x^6 - x^4 - x^3 - 1 = 0$ has one real root between 1.4 and 1.5. Find the root to 4 decimal places by false position method.

20. Apply Newton's method to obtain the real root of the equation $x \log_{10} x = 4.7772393$, correct to 6 places of decimals.

21. Find the root of the equation $xe^x = 3$ using the regula-falsi method correct to 4 decimal places.

22. Find a positive real root of $x \log_{10} x = 1.2$ using bisection method.

23. Find by Newton's method, the real root of the equation $3x = \cos x + 1$, correct to 4 decimal places.

24. Find a root of the equation $x^3 - 2x - 5 = 0$ using secant method correct to 3 decimal places.

25. Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$ correct to 5 decimal places.

26. Starting with $x = 0.12$, solve $x = 0.21 \sin(0.5 + x)$ by using iterative method.

27. Obtain the more general formula for the root of $f(x) = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} - \frac{1}{2}\frac{\{f(x_0)\}^2 f''(x_0)}{\{f'(x_0)\}^3}.$$

28. Show that the modified Newton-Raphson method

$$x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n)}$$

gives a quadratic convergence when the equation $f(x) = 0$ has a pair of double roots in the neighbourhood of $x = x_n$.

29. Show that following two sequences, both have convergence of the second order with the same limit $\sqrt{a}$.

$$x_{n+1} = \frac{1}{2}x_n\left(1 + \frac{a}{x_n^2}\right) \text{ and } x_{n+1} = \frac{1}{2}x_n\left(3 - \frac{x_n^2}{a}\right).$$

30. Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the method of regula-falsi to determine it.

31. Find a real root by iteration method of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0, 1]$ with an accuracy of 10^{-4}.

32. Prove that the order of convergence of the Newton-Raphson's method is 2.

33. Find a real root of the equation $x \log_{10} x = 1.2$ by regula-falsi method correct to 4 decimal places.

34. Solve the equations $x = x^2 + y^2$, $y = x^2 - y^2$ using Newton-Raphson method with the approximation $(0.8, 0.4)$.

35. Use Newton-Raphson method to solve the equations $x^2 - y^2 = 4$, $x^2 + y^2 = 16$ with $x_0 = y_0 = 2.828$.

36. Find a root of the system of nonlinear equations by Newton-Raphson method, $x^2 + y = 11$, $y^2 + x = 7$ with $x_0 = 3.5$ and $y_0 = -1.8$.

37. Solve the system of equations $\sin xy + x - y = 0$ $y \cos xy + 1 = 0$ with $x_0 = 1$ and $y = 2$, by Newton-Raphson's method.

38. Find the root of $\tan x + x = 0$ upto 2 decimal places, which lies between 2 and 2.1.

39. Show that the order of convergence of the secant method is 1.618 approximately.

40. Determine the order of convergence of the regula-falsi method. (Answer : 1.618, same as the order of convergence of the secant method.)

41. Use the method of false position, to find the fourth root of 32 correct to 3 decimal places.

42. Find the roots of the equation
$$2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$$
which has 2 roots greater than -1. Find these roots correct to 5 decimal places.

43. By using the Newton-Raphson method, find a root of the equation $1 - \cosh x . \cos x = 0$, correct to 4 decimal places, with $x_0 = 0.15$ as an initial approximation.

44. Using bisection method, find the negative root of the equation $x^2 + \cos x - 2 = 0$.

45. Using the Newton-Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. Carry out 4 iterations. Here x is in radians.

46. Newton-Raphson method for solving the equation $f(x) = c$, where c is a real-valued constant, is applied to the function
$$f(x) = \begin{cases} \cos x & \text{when } |x| \leq 1 \\ \cos x + (x^2 - 1)^2 & \text{when } |x| \geq 1. \end{cases}$$
For which c is $x_i = (-1)^i$, when $x_0 = 1$, and the calculation is carried out with no error 1.

47. Verify the equation
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} e^{0.3x}$$
has a root between 2 and 3. Find this root correct to 3 decimal places.

48. Solve $x^2 + 3x - y - 1 = 0$ $xy + 3y + 9 = 0$ starting with the approximation $x_0 = -4, y_0 = 6$.

49. Solve $x = 1 + \tan^{-1} x$ by the method of iteration.

50. By the fixed point iteration process, find the root correct to 3 decimal places, of the equation $x = \cos x$ near $x = \pi/4$.

51. Using bisection method, find an approximate root of the equation $\sin x = 1/x$, that lies between $x = 1$ and $x = 1.5$ (measured in radians). Carry out computations upto 7th stage.

52. Solve the Keplar's equation iteratively for $m = 0.8, E = 0.2$ by writing in the form $x = m + E \sin x$ and starting with $x_0 = m = 0.8$.

53. Find a double root of the equation $f(x) = x^3 - x^2 - x + 1 = 0$.

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